Model Reduction Techniques for the Analysis and the Design of Large-Scale Electromagnetic Devices

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In the analysis and design of large-scale dynamical systems, simpler models are often preferred to full system models due to their better suitability with computer simulations and real-time constraints. Model reduction techniques aim at yielding a reasonable trade-off between the contrasting needs of reducing the number of states and of reaching a good approximation of the overall system behavior. Moreover, in the specific case of complex electromagnetic devices (as fusion machines) a large number of state variables represent physical quantities in the overall system. This work collocates along this line of research and aims at studying Model Order Reduction techniques that maintain the mathematical formalism of system theory but at the same time keep consistency with the physics of the phenomena of interest throughout the whole reduction procedure and with respect to the final reduced order models.

***Index Terms*—Model Order Reduction, Selective Modal Analysis, Large-Scale Electromagnetic Modeling, Dynamical Systems**

I. Introduction

In the field of Computational Electromagnetics (CEM), modeling and simulation can be considered an approach for both the analysis and the synthesis of physical systems that complements theory and experiment (see for example, [1], [2]). The mathematical model can be obtained by several thousands of equations synthesized, for example, from a 2D or 3D finite element approach. In particular, the employment of such models in the context of fusion plasma science and engineering has become of paramount importance. Fusion devices are complex electromagnetic (EM) machines, whose behaviors depend on several controllable and non-controllable interconnected subsystems such as the plasma, the electromagnetic circuits, the passive conducting structures, the diagnostics. The coupling of these elements inherently leads to high order, non-linear systems to regulate, and there is consensus in the fusion community that active control will be one of the key enabling technologies, whence the need of accurate models derives [3].

Nonetheless, the straigthforward employment of these models translates into heavy computational burden, unfeasible execution times and the arising of numerical issues, thus calling for the employment of Model Order Reduction (MOR) techniques that allow to obtain reduced models by *controlling* the approximation procedure (Fig. 1). Actually, when deciding on the model complexity, a compromise must be reached between the detail of the representation of the phenomena of interest and the model complexity.

II. Linear MOR for Large Scale EM Models

For the scope of this work, it is useful to restrict the attention to linear, time invariant systems $\Sigma = \{A, B, C, D\}$ of order n characterized by a transfer function $G(s)$. The task of the MOR procedure is to compute a reduced system of order $r << n$, $\Sigma_r = \{A_r, B_r, C_r, D_r\}$ with transfer function $G_r(s)$, that approximates Σ according to some metrics in the time domain (w.r.t. the input-output or to the state behaviors) or in the frequency domain (w.r.t. the system frequency response).

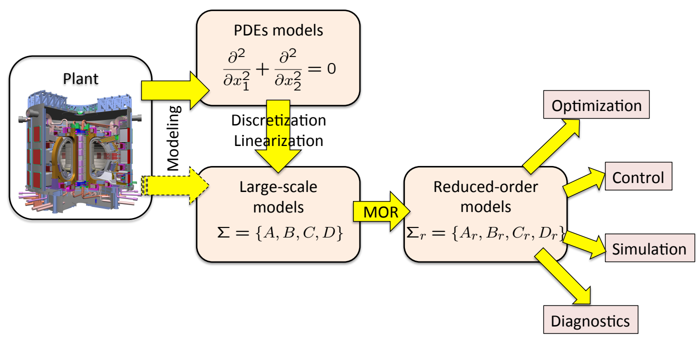
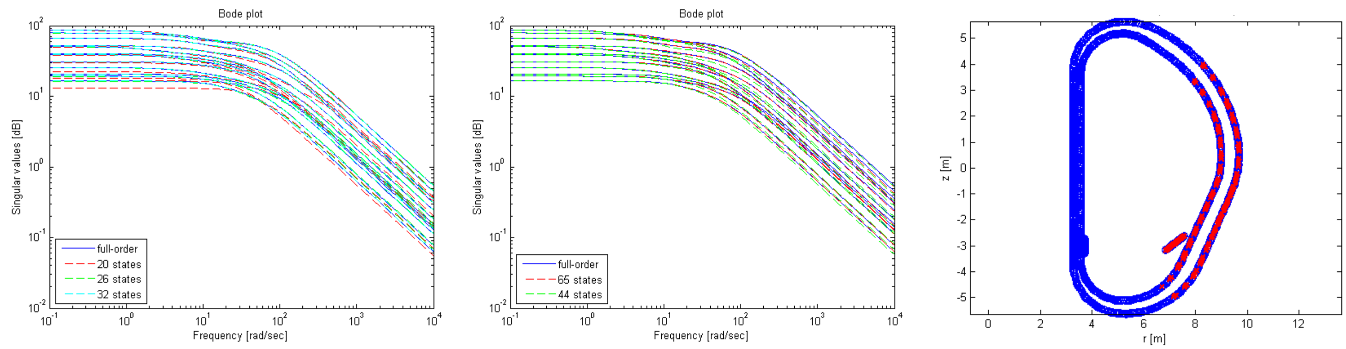


Fig. 1. Main steps in the study of a complex dynamical system. The final employment of the reduced order models regards plant optimization and control design, numerical simulation and software-in-the-loop diagnostic tools.

Linear MOR of dynamical systems can be implemented by exploiting numerous approaches, such as modal truncation, Gramian-based reduction techniques (Truncated Balanced Realization (TBR), optimal Hankel norm approximation), Krylov-subspace projection, proper orthogonal decomposition reduction [4]. All these are popular methods based on space projection, but because of the change of basis over the space state they have the drawback of losing the physical meaning of the starting model variables.

In the specific case of large EM devices (as fusion machines) a large number of state variables is linked to active (magnetic field sources) and passive (conducting parts surrounding the plasma) structures, modelled by mesh elements in the geometrical discretization of the numerical domain adopted either in finite elements models (FEMs) or integral codes. Since these states represent physical quantities in the overall system, such as currents and voltages, it would be important not to loose this valuable feature while reducing the order of the system.

The scope of this work is to exploit other methods such as Selective Modal Analysis (SMA), which allows to preserve



(a) TBR procedure (b) Krylov-based MOR (c) SMA - participation ratio

Fig. 2. Results of the MOR procedure. (a)-(b) TBR vs Krylov: principal singular value comparison between Σ full-order with $n = 110$ and Σ_r with r as indicated; (c) SMA: the states that are most contributing to the eigenmodes (states related to the first mode are shown) are selected in the SMA procedure. Large values of the participation ratio mean that the selected states give a contribution to the i-th mode greater than the contribution of the neglected ones.

this meaning resorting to a state selection according to the contribution of the single states to the system modes [5] and to apply these methods to large scale FEM models.

III. Simulation Results

The application of the MOR approach to the models of the ITER machine [6] with hundreds/thousands states aims at producing accurate and compact linear models (e.g. tens of states) that could be efficiently coupled with real time reconstruction codes [7] for diagnostics and control purposes.

In this respect, to exemplify this approach, two full-order models with $n = 1832$ and $n = 110$ states are considered, where these states are derived in the former case from a 2D simplicial mesh and in the latter case from a geometry based discretization of the passive conductive structures.

Both related systems Σ appear to be stable, controllable and observable, and so they are minimal realizations of their transfer function. The analysis of the Hankel singular values highlights how most of the information is contained in the first 30–40 states of the systems and it is interesting to note how this value corresponds to the *degrees of freedom* of the operational space of the ITER machine. Comparing the different techniques (Fig. 2(a)-(b)), it can be observed how TBR and Hankel norm approximation attain huge model reduction at the cost of losing the physical meaning of the retained states; similarly, the Krylov approach provides a computationally efficient method (through the Arnoldi algorithm) but with a lower reduction rate.

Finally, the SMA introduces the participation ratios that can be employed as a guideline to choose states and order of the reduced model (Fig. 2(c)). In detail, the participation factor p_{ki} measures the contribution of the k-th state in the i-th mode. It is defined as the product of the k-th components of the left and right normalized eigenvector (w_{ki} and v_{ki} respectively) corresponding to the i-th mode; the participation ratio ρ_{ri} measures the overall contribution of a set of r states:

$$p_{ki} := w_{ki} v_{ki} \Rightarrow \rho_{ri} := \frac{\sum_{k=1}^{r} p_{ki}}{\sum_{k=r+1}^{n} p_{ki}}. \qquad (1)$$

Moreover, the analysis of the ratios $\{\rho_{ri}\}$ yields an interesting interpretation of the modal pattern of the structure response. It can be observed, and it will be shown in the full paper, that:

- low-frequency modes depend on states that typically present symmetries with respect to radial or vertical axes;
- middle-frequency modes depend on states that are evenly distributed in the metallic structures;
- high-frequency modes behave similarly to the middle-frequency ones, except for the very high-frequency modes, which appear to be dependent on localized currents without any symmetric pattern.

To select which states can be neglected while retaining the most relevant dynamic behavior of the system, a typical choice for a stable system is to keep the most influential variables on the low frequency eigenvalues of the system (i.e. the dominant ones). In this sense, to achieve good performances in approximating the full-order system dynamics, a higher number of states must be selected respect to the other MOR techniques.

IV. Conclusion

In the analysis of large-scale EM systems, MOR aims at yielding a reasonable trade-off between the contrasting needs of reducing the number of states to provide compact models and of reaching a good approximation of the overall system behavior. Since the full system order can be large it is important to develop procedures that attain not only a *mathematically efficient* order reduction as with classical MOR techniques, but also a *physically consistent* approach so as to simplify the system understanding and ease the controller design.

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References

[1] D.A.Lowther, "The Development of Industrially-Relevant Computational Electromagnetics Based Design Tools", *IEEE Trans. Magn.*, vol. 49(5), pp. 2375-2380, 2013.

[2] A.Demenko, R.M.Wojciechowski, J.K.Sykulski, "2-D Versus 3-D Electromagnetic Field Modeling in Electromechanical Energy Converters", *IEEE Trans. Magn.*, vol. 50(2), pp. 897-900, 2014.

[3] —, "Special section on Control of tokamak plasmas: Part I and II", *IEEE Control Syst. Mag.*, Part I: vol.25(5), 2005; Part II: vol.26(2), 2006.

[4] A.Antoulas, *Approximation of Large-Scale Dynamical Systems*, SIAM series on Advances in Design and Control, 2009.

[5] A.Beghi, "An application of selective modal analysis to tokamak modeling and control", *IEEE Trans. Control Syst. Technol.*, vol.9(4), pp.574-589, 2001.

[6] —, ITER website, http://www.iter.org, [Online, April 2015].

[7] P.Bettini and A.Cenedese, "Iterative Axisymmetric Identification Algorithm (IAIA) for real-time reconstruction of the plasma boundary of ITER", *Fus. Eng. and Des.*, vol. 88(68), pp.1150-1155, 2013.